Iterative Volumetric ECC Schemes for Holographic Data Storage

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ABSTRACT

A new three-dimensional error correction coding (ECC) scheme for holographic data storage, or iterative volumetric ECC (IVECC), and its decoding algorithm have been developed and evaluated. The IVECC constructs volumetrically coupled ECC blocks, which can correct errors for three dimensions, leading to favorable ECC performance in page-based recording and retrieving system. It also can recover data of seriously corrupted pages or even missing pages by iterative decoding, may occur due to imperfect recording media or other defects on optical devices in holographic data storage.

Keywords: Holographic data storage, page of memory, error correction code, Reed-Solomon code

1. INTRODUCTION

Holographic digital data storage (HDDS) system is a page-oriented digital data storage paradigm and has been investigated for the next generation of digital storage owing to its ability to provide high data transfer rate and high density.[1, 2] High data transfer rate is enabled by the writing and reading of many bits in parallel in the form of large pixelated data pages and high density derives from the ability to multiplex many holographic pages within the same small storage volume. Like a digital communication system, data that are retrieved form a HDDS are usually impaired by various noise sources such as optical scatter, electronic noise, and inter-symbol interference (ISI), of which effects create local variations in the intensity levels within a retrieved page. In addition, since data pages are recorded and read by page unit with holographic principle, the signal qualities such as signal to noise ratio (SNR) or raw bit error rate (BER) varies among retrieved pages, even greatly varies when adapting locally inhomogeneous recording media or less reliable image aligning servo, considering the limited cost for custom product or high speed operation. Those undesired serious corruptions are rare but unavoidable, which can yield decoding errors with commonly used error correction coding (ECC) schemes,[3] such as Reed-Solomon (RS) code or RS product code (RSPC). Those conventional schemes can correct only limited number of errors in a page that fail to reconstruct data with a fixed coding overhead when there exist severely corrupted blocks, although overall raw symbol error rate (SER) is controllable with that overhead. Recently, three dimensional (3D) interleaver[4] is presented for distributing volume cluster errors to randomly distributed errors, which can be applied to mitigate negative effects of those variations among pages with large degree of interleaving, covering multiple pages. In addition, 3D array ECC scheme[5] are presented for page-based data storage system, however, of which error correcting capabilities are too limited to cover varying errors of pages with fixed sets of array ECC codes. In this paper, the Iterative Volumetric error correcting code, or IVECC, is proposed to correct different level of error rates of pages, efficiently and powerfully. It constructs volumetrically coupled ECC blocks by additional ECC encoding of two dimensionally ECC encoded pages like RSPC blocks, which makes it possible to correct errors for three dimensions, leading to favorable ECC performance in a page-based recording and retrieving system. By a number of iterations of ECC decoding, the proposed IVECC also can recover data of seriously corrupted pages or even missing pages, may occur due to imperfect recording media or other defects on optical devices in holographic data storage, without any interleaver having large degree of interleaving. The results of numerical simulations and actual recording experiments are presented to verify the performances of proposed ECC scheme comparing to conventional ones, constructed with commonly used RS ECC code. Additionally, for practical implementation in the field, processing delay and hardware complexity

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are also investigated with hardware level simulation and synthesis. The results show that IVECC gives favorable error correcting performance in page-based holographic data storage system with slight increase of hardware complexity.

2. ITERATIVE VOLUMETRIC ECC CODING SCHEME

In a data storage system, the goal of ECC coding is to reduce the bit error rate (BER) to sufficiently low level, while achieving such important merit as high density and high data transfer rate. Although the system retrieves raw data from the storage device with many errors (i.e. high raw BER), the ECC ensures that the user data is delivered with an acceptably low error level (i.e. low user BER). The proposed ECC scheme can efficiently control those errors by adapting three dimensional ECC encoding as well as iterative ECC decoding for each direction successively, as follows.

2.1. IVECC Encoding

IVECC encoding consists of two operations: Product ECC encoding for two-dimensional pages of data such as RSPC, and additional Page ECC encoding to combine pages together with another ECC code. Let **D** and **C**, a block of input data and encoded data, respectively, be a three-dimensional array. A three-dimensional input data block **D** and encoded data block **C** are abbreviated as

$$\mathbf{D} = [d_{\alpha\beta\gamma}]_{k_1\times k_2\times k_3} \quad (1)$$

$$\mathbf{C} = [c_{\alpha'\beta'\gamma'}]_{n_1\times n_2\times n_3} \quad (2)$$

where $d_{\alpha\beta\gamma}$ and $c_{\alpha'\beta'\gamma'}$ are m-bit integer symbol. n_1, n_2 and n_3 are lager than k_1, k_2 and k_3, respectively, and less than $2^m - 1$. An input data block **D** consists of k_3 two-dimensional array $\mathbf{D}_i$ of area $k_1 \times k_2$, and encoded data block **C** is composed of n_3 two-dimensional $n_1 \times n_2$ ECC code array $\mathbf{C}_j$. This three-dimensional arrays are constructed by stacking several two-dimensional array. Each two-dimensional array is represented by

$$\begin{aligned}
\mathbf{D}_i &= \begin{bmatrix} d_{11i} & d_{12i} & \cdots & d_{1k_2i} \\ \vdots & \vdots & \vdots & \vdots \\ d_{k_11i} & d_{k_12i} & \cdots & d_{k_1k_2i} \end{bmatrix} \quad (i = 1, 2, \cdots, k_3) \\
&= [d_{\alpha\beta\gamma}]_{k_1\times k_2\times 1}\big|_{\gamma=i} \\
&= \begin{bmatrix} {}^i\underline{d}_1^{(r)T} & {}^i\underline{d}_2^{(r)T} & \cdots & {}^i\underline{d}_{k_1}^{(r)T} \end{bmatrix}^T \quad (3) \\
&= \begin{bmatrix} {}^i\underline{d}_1^{(c)} & {}^i\underline{d}_2^{(c)} & \cdots & {}^i\underline{d}_{k_2}^{(c)} \end{bmatrix} \quad (4) \\
\mathbf{C}_j &= \begin{bmatrix} c_{11j} & c_{12j} & \cdots & d_{1n_2j} \\ \vdots & \vdots & \vdots & \vdots \\ c_{n_11j} & c_{n_12j} & \cdots & c_{n_1n_2j} \end{bmatrix} \quad (j = 1, 2, \cdots, n_3) \\
&= [c_{\alpha'\beta'\gamma'}]_{n_1\times n_2\times 1}\big|_{\gamma=j} \quad (j = 1, 2, \cdots, n_3) \\
&= \begin{bmatrix} {}^j\underline{c}_1^{(r)T} & {}^j\underline{c}_2^{(r)T} & \cdots & {}^j\underline{c}_{n_1}^{(r)T} \end{bmatrix}^T \quad (5) \\
&= \begin{bmatrix} {}^j\underline{c}_1^{(c)} & {}^j\underline{c}_2^{(c)} & \cdots & {}^j\underline{c}_{n_2}^{(c)} \end{bmatrix} \quad (6)
\end{aligned}$$

where ${}^i\underline{d}_\alpha^{(r)}$ and ${}^i\underline{d}_\beta^{(c)}$ are row and column vector of *i-th* data array of input data block **D**,respectively. Similarly, ${}^j\underline{c}_{\alpha'}^{(r)}$ and ${}^j\underline{c}_{\beta'}^{(c)}$ are row and column vector of *j-th* code array of encoded data block **C**.

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Product ECC : Product ECC encodes each two-dimensional input array $\mathbf{D}_i$ to $n_1 \times n_2$ encoded array $\mathbf{C}_j$, Product ECC encoder apply error correction code for each row and column consecutively, generate parity checks along the rows and columns of $k_1 \times k_2$ input data bit array, and then, construct $n_1 \times n_2$ code array $\mathbf{C}_i$ by parity check. By applying specific ECC coding to k_1 row vectors ${}^i\underline{d}_\alpha^{(r)}$ of *i-th* input data array $\mathbf{D}_i$, row vector ${}^i\underline{m}_\alpha^{(r)}$ of n_2 dimension is given. Then, k_1 row vector ${}^i\underline{m}_\alpha^{(r)}$'s are compose of two-dimensional ECC array $\mathbf{M}_i$ with $k_1 \times n_2$ dimension.

$$\begin{aligned} \mathbf{M}_i &= [{}^i\underline{m}_1^{(r)T}\ {}^i\underline{m}_2^{(r)T}\ \cdots\ {}^i\underline{m}_{k_1}^{(r)T}]^T \\ &= [{}^i\underline{m}_1^{(c)}\ {}^i\underline{m}_2^{(c)}\ \cdots\ {}^i\underline{m}_{n_2}^{(c)}] \end{aligned} \tag{7}$$

where $\underline{m}_{ip}^{(r)} \in \mathbb{Z}^{n_2}$ $(p = 1, 2, \cdots, k_1)$ and $\underline{m}_{ip}^{(c)} \in \mathbb{Z}^{k_1}$ $(q = 1, 2, \cdots, n_2)$

By applying another specific ECC to n_2 column vectors ${}^i\underline{m}_{\beta'}^{(c)}$, column vector ${}^j\underline{c}_{\beta'}^{(c)}$ $(j = i,\ \beta' = \beta)$ with n_1 dimension. Therefore, we obtain two-dimensional code array $\mathbf{C}_i$ from data page $\mathbf{D}_i$.

Page ECC : Page ECC follows Product ECC. Let $\mathbf{L}$ be a three-dimensional array, piled up k_3 Page ECC encoded array $\mathbf{C}_j$

$$\begin{aligned} \mathbf{L} &= [c_{\alpha'\beta'\gamma'}]_{n_1 \times n_2 \times k_3} \\ &= \begin{bmatrix} \underline{l}_{11}^{(l)} & \underline{l}_{12}^{(l)} & \cdots & \underline{l}_{1n_2}^{(l)} \\ \vdots & \vdots & \vdots & \vdots \\ \underline{l}_{n_1 1}^{(l)} & \underline{l}_{n_1 2}^{(l)} & \cdots & \underline{l}_{n_1 n_2}^{(l)} \end{bmatrix} \end{aligned} \tag{8}$$

where $\underline{l}_{\alpha'\beta'}^{(l)}$ is layer vector with k_3 dimension. Page ECC encodes each $\underline{l}_{\alpha'\beta'}^{(l)}$ to layer vector of n_3 dimension of encoded data block $\mathbf{C}$ with additional ECC. Finally, we obtain the IVECC encoded block $\mathbf{C}$ of $n_1 \times n_2 \times n_3$.

Example : We present an example with RS ECC code, which just concatenates check symbols for ECC to input data symbols. For a block of input data $\mathbf{D}$ of $k_1 \times k_2 \times k_3$ dimensions of which elements are m-bit symbols. At first, Product ECC or RSPC, is applied to each row and column vectors of $\mathbf{D}_i$'s. Using RS $(n_2,\ k_2)$, RSPC encodes each row vectors ${}^i\underline{d}_\alpha^{(r)}$ of data page to n_2 dimensional code word ${}^i\underline{m}_\alpha^{(r)}$ which $n_2 - k_2$ parity symbols to each row vector ${}^i\underline{d}_\alpha^{(r)}$

$${}^i\underline{m}_\alpha^{(r)} = [{}^i\underline{d}_\alpha^{(r)} | {}^i\underline{p}_\alpha^{(r)}] \tag{9}$$

where ${}^i\underline{p}_\alpha^{(r)} \in \mathbb{Z}^{n_2 - k_2}$ is row check symbol vector. As shown in (7), k_1 row vectors compose of $k_1 \times n_2$ array $\mathbf{M}_i$. And then, for a n_2 column vectors of $\mathbf{M}_i$, it encodes each column to n_1 dimensional code word ${}^i\underline{c}_{\beta'}^{(c)}$ with RS (n_1,k_1). It can be seen that ${}^i\underline{c}_{\beta'}^{(c)}$ becomes

$${}^i\underline{c}_{\beta'}^{(c)} = \begin{bmatrix} {}^i\underline{m}_{\beta'}^{(c)} \\ \hline {}^i\underline{p}_{\beta'}^{(c)} \end{bmatrix} \tag{10}$$

where ${}^i\underline{p}_{\beta'}^{(c)}$ is column check symbol vector of $n_1 - k_1$ dimensions.

Consequently, $\mathbf{C}_i$ is represented as

$$\mathbf{C}_i = \begin{bmatrix} \mathbf{D}_i & \mathbf{P}_i^{(r)} \\ \mathbf{P}_i^{(c)} & \mathbf{P}_i^{(p)} \end{bmatrix} \tag{11}$$

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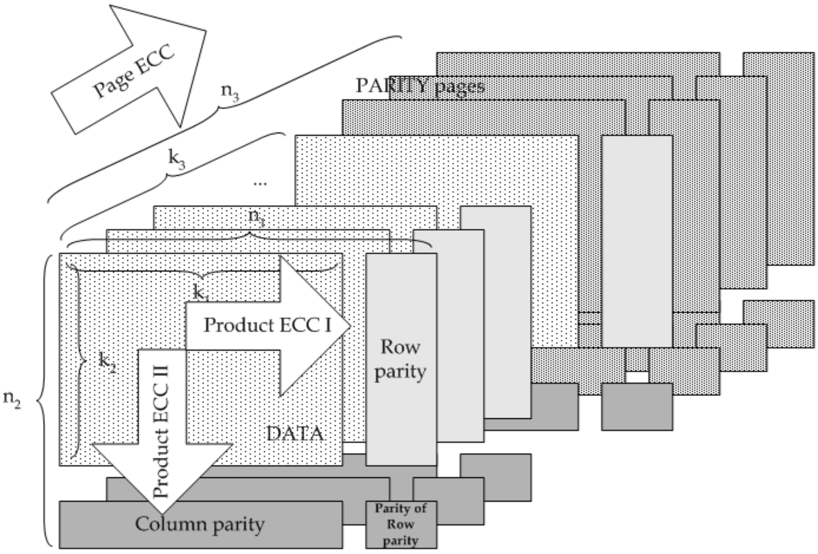

Figure 1. Iterative Volumetric ECC encoding.

where $\mathbf{P}_i^{(r)}$ and $\mathbf{P}_i^{(c)}$ are row and column parity array for i-th input data array $\mathbf{D}_i$. $\mathbf{P}_i^{(p)}$ is parity array of i-th row parity array.

Then, finally, for k_3 sets of those $\mathbf{C}_i$'s, additional ECC encoding, such as RS (n_3, k_3),is applied to each layer vectors. Each layer vector $\underline{c}^{(l)}_{\alpha'\beta'}$ of three-dimensional code block $\mathbf{C}$ is obtained by

$$\underline{c}^{(l)}_{\alpha'\beta'} = [\underline{l}^{(l)}_{\alpha'\beta'} \mid \underline{p}^{(l)}_{\alpha'\beta'}] \tag{12}$$

Fig. 1 shows schematics of encoding procedure of IVECC. As you can see, one of the key isues of proposed method is additional ECC check pages valuable to improve error correction performance significantly with iteration, which will be discussed in following section.

2.2. IVECC decoding

The decoding procedure is number of repetitions of two steps : Page ECC decoding and Product ECC decoding, of which orders are changeable. Let $\widehat{\mathbf{C}}$, a 3D block of retrieved data, be a three-dimensional array consisting of $n_1 \times n_2$ layer vectors $\widehat{\underline{c}}^{(l)}_{\alpha'\beta'}$ of n_3 dimensions, where $\widehat{\underline{c}}^{(l)}_{\alpha'\beta'} = [\widehat{c}_{\alpha'\beta'\gamma'}]_{1\times1\times n_3}$, and $\widehat{c}_{\alpha'\beta'\gamma'}$'s are retrieved m-bit integer symbols. At first, Page ECC decoding is applied to each vector $\widehat{\underline{c}}^{(l)}_{\alpha'\beta'}$, which corrects errors of some of them that specific ECC coverable, and leaves others. Then, the Product ECC decoding is applied to the first k_3 pages $\widehat{\mathbf{C}}_j$ $(j = 1, 2, \cdots, k_3)$ of retrieved data block, which also corrects errors of some of column vectors ${}^{j}\widehat{\underline{c}}^{(c)}_{\alpha'}$'s, that column ECC coverable, and corrects errors of some of row vectors ${}^{j}\widehat{\underline{c}}^{(r)}_{\beta'}$'s, that row ECC coverable, sequentially. The output of first turn of two steps, noted to $\widehat{\mathbf{C}}^{(1)}$, might have errors that could not be corrected due to the limited correctible number of symbols for specific ECC. Then, second applying of Page ECC to $\widehat{\mathbf{C}}^{(1)}$ can correct errors of some vectors that could not corrected in the first attempts, with the help of Product ECC, which enabled those by correcting some of errors. In the same way, the second applying of Product ECC also can correct errors of some column or row vectors as a result of second Page ECC. Therefore, iterations of those two steps can significantly reduce error rates of the retrieved data, of which the number depends on the length of check symbols in the data block. After iterating r times, removing check symbols in $\widehat{\mathbf{C}}^{(r)}$, reconstructed user data block $\widehat{\mathbf{D}}$ is given with reliably low error rates. In this way, IVECC can control errors efficiently, even in the presence of severely corrupted pates or missing pages, where product ECC only might be saturated with errors.

Example : The decoding procedures of previous evaluation with RS ECC code will be described.

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1. Page ECC decoding,or RS(n_3,k_3) decoder, is applied to the vectors $\hat{\underline{c}}^{(l)}_{\alpha'\beta'}$'s of $\widehat{\mathbf{C}}$

2. Correct some of vectors, having less number of errors than $(n_3 - k_3)/2$ and update $\widehat{\mathbf{C}}$.

3. Product ECC decoding of RSPC is applied to each two-dimensional array $\widehat{\mathbf{C}}_i$ of recently updated retrieved data block $\widehat{\mathbf{C}}$.

4. Correct errors of each column vectors ${}^{j}\hat{\underline{c}}^{(c)}_{\beta'}$ of $\widehat{\mathbf{C}}_j$,having less number of errors than $(n_2 - k_2)/2$, with (n_2,k_2) RS and update each vectors.

5. Correct errors of each row vectors ${}^{j}\hat{\underline{c}}^{(r)}_{\alpha'}$ of $\widehat{\mathbf{C}}_j$,having less number of errors than $(n_1 - k_1)/2$, with (n_1,k_1) and update each vectors.

6. Repeat above steps.

The error correction with RSPC affects the next stage that some layer vector $\hat{\underline{c}}^{(l)}_{\alpha'\beta'}$'s, unable to correct in the previous stage due to lager number of errors that $(n_3 - k_3)/2$, has changed to be correctible. In this same way as a result of Page ECC, RSPC decoder also have some correctible row or column vectors, which are repeated several times for data retrieving with reliably low error level. Then, the user data vectors, can be acquired by removing $n_1 - k_1$ checks symbols from

3. VERIFICATIONS

In order to verify error correcting performances of the proposed scheme, numerical simulations and actual recording experiments have been conducted with RS ECC code, which is widely used for optical data storage. RS ECC and RSPC ECC coding schemes are also evaluated with scheme, noted RSVC (RS Volumetric Code), in the same error conditions with same code rate.

3.1. Numerical simulation

For numerical simulation, the two sets of ECC schemes, 0.90 and 0.95 code rates, are evaluated with RS ECC code with 8 bit symbol as listed Table I. The error patterns, randomly distributed errors with various error injection rates, are generated for pages of 360 by 360 dimensions of 8-bit symbol, corresponds to one mega-bit pages. For a certain error injection rates, a set of error patterns of 10^5 pages are tested, of which variations are controlled to have 10% of mean error injection rate with Gaussian distribution, in order to include the effects of SER variations of retrieved page in holographic data storage channel.

Table 1. ECC setups for numerical simulation

ECC code rate	RS ECC only	RSPC ECC	RSVC ECC
$\sim$ 0.90	RS(238,214)	RS(238,224) RS(234,224)	RS(238,230) RS(234,224) RS(230,224)
$\sim$ 0.95	RS(238,226)	RS(238,230) RS(234,230)	RS(238,234) RS(234,230) RS(234,230)

For RSPC ECC decoding, the 3D volume interleaver[3] is also applied to distribute errors uniformly for pages with cluster size of 11 and degree of interleaving of 231. The corrected SER's for various ECC schemes of 0.90 and 0.95 code rate of the simulation are plotted as a function of mean error injection rates in Fig. 2 (a) and (b),

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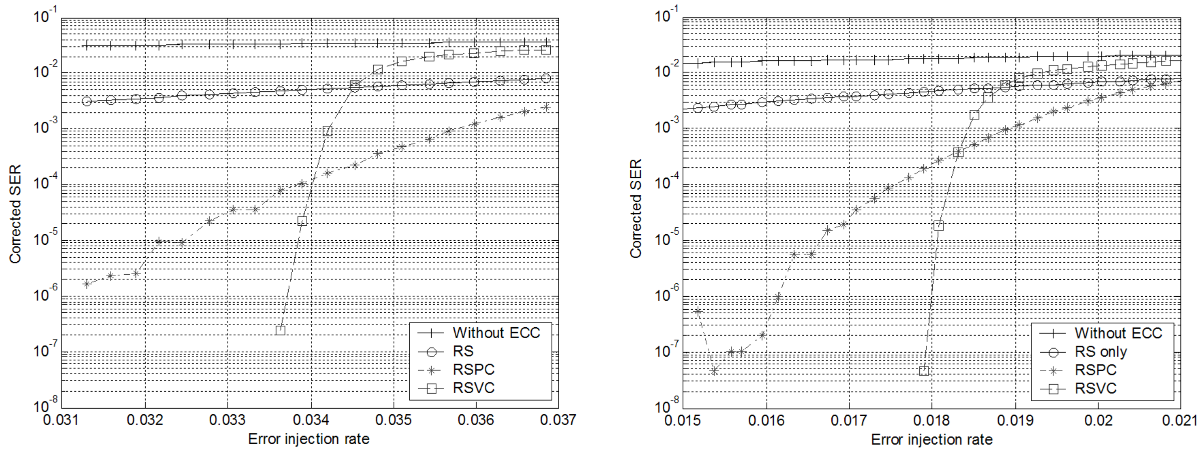

(a) (b)

Figure 2. Corrected SER's (Simulation) of various ECC schemes (a) 0.90 (b) 0.95 ECC code rate.

respectively. For the generated error patterns, RS(n_h,k_h) is made to correct all errors, if the number of errors in a codeword of length n_h is less than $(n_h - k_h)/2$, otherwise, no change is applied to the error patterns. For RSPC and RSVC, the above procedures are repeatedly applied for each direction, iteratively. The number of iterations is fixed to 5, which is determined by the iterative simulation results, to saturate corrected SER value with 95% accuracy. As you can see in Fig. 2, RSVC yields better error control performance than others in wide range, except for higher than critical point of error rates, improbable in normal conditions.

3.2. Experimental verification

Actual recording experiments have also been conducted to evaluate the proposed ECC scheme. The schematic diagram of the optical setup of experiments, Daewoo electronics quarter mega-pixel HDDS system (DE-QM), is shown in Fig. 3. An Nd-Yag laser with a maximum output power of 200 mW at 532 nm is used as a light source. The light beam from the laser is expanded by a high-power beam expander and is split into the signal arm and the reference arm with controlled modulation depth by a wave plate (WP) and a polarizing beam splitter (PBS). The signal beam is modulated by a photo-chrome MASK with 512 by 512 pixels of 18 m and is Fourier-transformed by a lens of 60 mm focal length. The charge-coupled device (CCD) camera is a SENSYS CCD camera with a

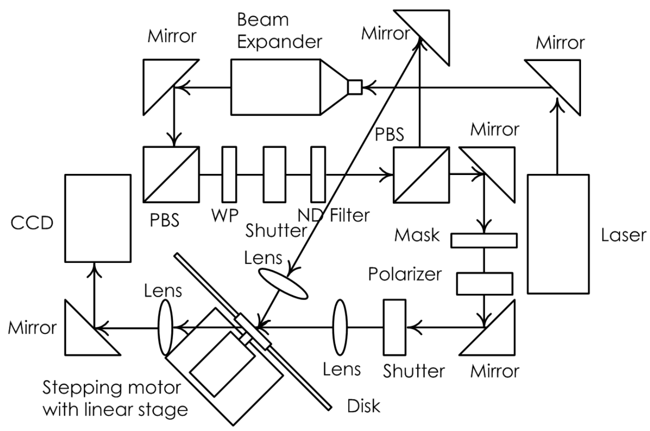

Figure 3. Schematics of the optical setup of experiments.

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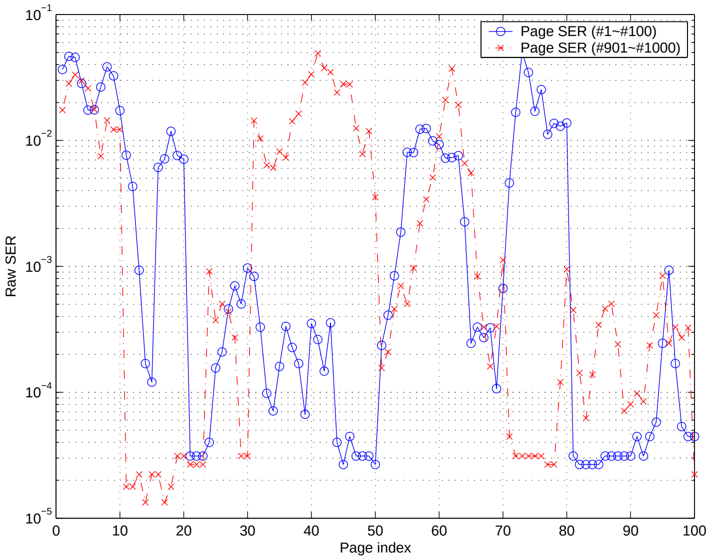

Figure 4. Raw SER of HDDS retrieved pages with shift-multiplexing technique.

1024×1024 resolution and is operated in 2×2 binning mode with a pixel matching configuration. The reference beam interferes with the signal beam in a photo-refractive polymer disk of $200\mu m$ thickness. In order to reduce the saturation of the recording media, we used defocusing of $5mm$ behind the focal plane of the lens. Holograms were recorded by spherical wave shift multiplexing with $100\mu m$ separation. An Oriental's stepping motor is used for this purpose. Sixteen different patterns of MASK of 512×512 512 size, which are modulated by several modulation schemes such as 6-8 balanced modulation,[2] 9-12 pseudo-balanced code (9-12 PBC),[6,8] codeword complementing block modulation (CCBM),[7] and simple global threshold to cover various error patterns.

The first experiment is for checking the raw SER variations of retrieved pages in DE-QM system. One thousand holograms, a quarter-megapixel pages of 480 by 480 channel-bits, corresponding to 28.8K symbols, were recorded and retrieved by capturing images from rotating disk. In order to reduce the pixel misalignment effects during retrieving, over-sampling technique, choosing only one optimum pixel in 2 by 2 over-sampled pixels is applied. Various modulation schemes are also applied for the retrieved images to reconstruct digital data, which are converted to 8-bit symbol arrays and compared to the original ones. Fig.4 shows raw SER's of each page for the first hundred pages. As seen in the figure, the SER values are varying wide range and, in particular, several pages are degraded severely refer to the high SER, which might be caused by blur effect due to defects, dust, or local non-uniformity of recording media, and detector pixel mis-registration or flaws on other optical components. In order to correct all the above page errors with conventional ECC, strong ECC codes are necessary due to the severely corrupted pages, which will increase overall ECC overhead, significantly. In order to relieve those variations of page SER, specially designed interleaver[3] used in simulation, is applied to the retrieved data. The distributions of page SER's are plotted in Fig. 5, which shows that the interleaver can successfully reduce the effects of page variations, while the variations are still not negligible. For the measured error patterns, various ECC schemes of different code rate are applied to compare error correction performances. For fast calculation, error correction of RS is applied in the same manner, as applied to the simulation ones. The corrected SER's of experimental results are also in Fig. 6, which shows that the proposed algorithm, RSVC in this example, outperforms previous ECC coding schemes in conjunction with the proposed volumetric coupling and iteration, in view of error control performance with limited code rate.

4. CONCLUSION

A new three-dimensional error correction scheme, or volumetric ECC with an iterative decoding, has been developed and evaluated. The Volumetric ECC can correct errors of even seriously degraded pages with two

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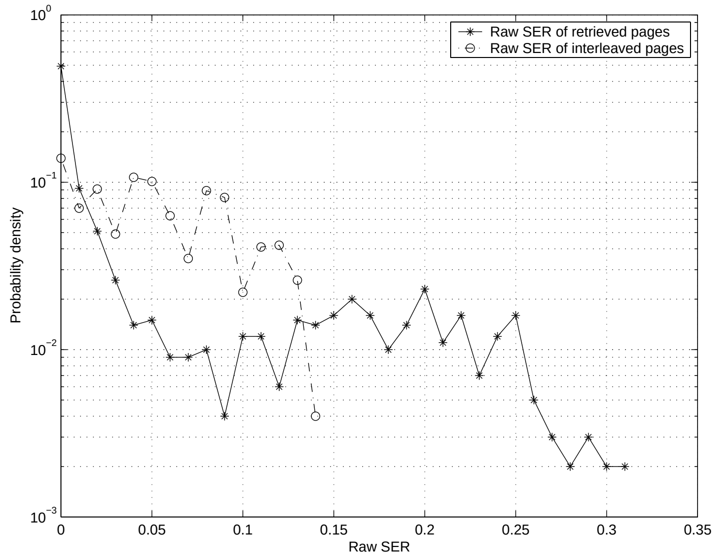

Figure 5. Page SER distributions of retrieved and interleaved pages.

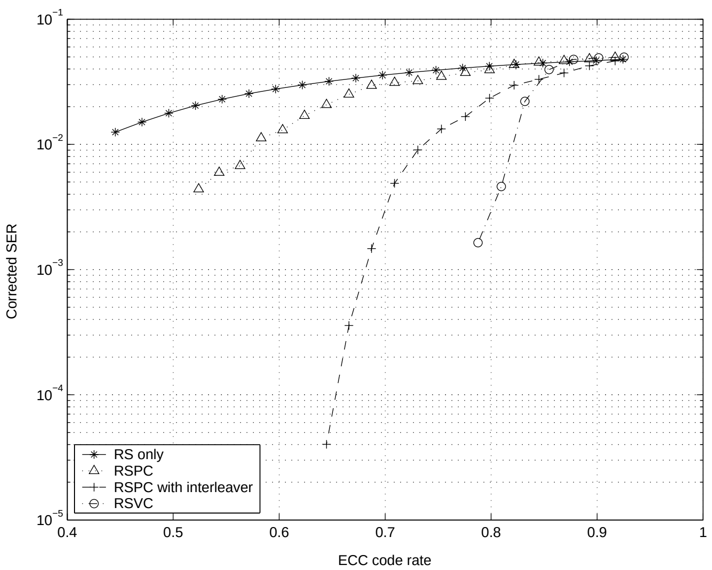

Figure 6. Corrected SER's with various ECC code rate (experiments).

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successive encoders, Product ECC and Page ECC, which shows better error control performance than previous algorithm. The results of numerical simulations and actual recording experiments show favorable error control capability of proposed IVECC scheme

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